## EXercises lecture 4

## EXERCISE 4.1

Calculate or determine the mass and the center of mass for the following objects.
i. A box with size ( $40 \mathrm{~cm}, 80 \mathrm{~cm}, 50 \mathrm{~cm}$ ) centered at $(-20 \mathrm{~cm}, 40 \mathrm{~cm}, 20 \mathrm{~cm})$ with uniform density of $1400 \mathrm{~kg} / \mathrm{m}^{3}$.
ii. A cylinder shell with inner radius 1 meter and outer radius 2 meters, aligned along the $x$-axis with length 40 cm and its center at $(20 \mathrm{~cm}, 20 \mathrm{~cm}, 20 \mathrm{~cm})$ with uniform density of $300 \mathrm{~kg} / \mathrm{m}^{3}$.
iii. The composite object consisting of the two objects above (i. and ii.).

## EXERCISE 4.2

Show that the moment of inertia in the z-direction $I_{z z}$ for a box with mass $m$ and dimensions ( $w, h, d$ ) is given by $I_{Z Z}=\frac{1}{12} m\left(w^{2}+h^{2}\right)$.

## ExERCISE 4.3

Show that the moment of inertia in the z-direction $I_{z z}$ for a cylinder oriented along the z-axis with mass $m$ and radius $r$ is given by $I_{z Z}=\frac{1}{2} m r^{2}$.

## EXERCISE 4.4

Calculate the moment of inertia along the x-axis $I_{x x}$ for the three objects of exercise 4.1. Please note that the moment of inertia of a cylinder shell along the $x$-axis is given by $I_{x x}=\frac{1}{2} m\left(r_{1}{ }^{2}+r_{2}{ }^{2}\right)$.

## EXERCISE 4.5

Given a ring with radius $R$ of $n$ cylinders, each having a mass of $M / n$, and a radius $r(\ll R)$. Determine the $I_{z z}$ for the entire system of $n$ cylinders. Remember that $I_{z Z}$ for a cylinder is $\frac{1}{2} m r^{2}$. What happens if $n$ becomes larger and larger?

## EXERCISE 4.6

Analyze the forces and torques on the box. What force $F_{p}$ is necessary to tip the box over?


## EXERCISE 4.7

Analyze the forces and torques on the rolling cylinder of radius $r$. What is its linear acceleration?


